

## The Ultimate Hull Girder Strength Analysis Considering Section Modulus Under Longitudinal Bending

*Muhammad Zubair Muis Alie and Samuel Izaak Latumahina*

Department of Naval Architect and Ocean Engineering, Engineering Faculty, Hasanuddin University  
Gowa, South Sulawesi, Indonesia

### ABSTRACT

The objective of the present study is to assess the ultimate hull girder strength taking the section modulus into account under longitudinal bending. A Ro-Ro Ship is taken as object ship. A Ro-Ro Ship has a unique character because most of the longitudinal elements locate above neutral axis. While there are not the longitudinal elements under the neutral axis particularly at the bottom part so that the bottom part consists plate only. The simple expression implemented into in-house program to calculate the section modulus of ship cross section is performed. The cross section is assumed to be remained plane and the simply supported of boundary condition is imposed on plate and stiffened plate elements in the cross section. The vertical bending moments are imposed to both sides of the cross section. The ultimate hull girder strength is calculated by considering the section modulus including their progressive collapse behavior for Ro-Ro ship hull.

**KEY WORDS:** Ship hull; cross section; section modulus; ultimate strength.

### INTRODUCTION

The Ro-Ro ship is one of the ship types which transport the cargo in horizontal direction and eliminate the need for onboard or deckside lift-on and/or lift off instrument. The Ro-Ro ship has been innovated to carry processed forest product, lumber, plywood, cars and many things. Ro-Ros are an important connection for the intermodal transportation network. In this regard, the section modulus could be one of the important parameters from the viewpoint of assessing the ultimate strength of all the decks on which cars, passengers and so on are put. In spite of human error, the structural degradation during loading and unloading gives impact to the ultimate strength of ship's hull.

The ultimate hull girder strength of merchant ships including Ro-Ro ship has been assessed by some researchers. Kukkanen, T and Matusiak, J (2014) presented the numerical and experimental results of nonlinear wave loads. A nonlinear time domain method had been developed and the theoretical background of the method were provided. The method was based on the source formulation expressed by means

of the transient three-dimensional Green function. The time derivative of the velocity potential in Bernoulli's equation was solved with similar source formulation to that of the perturbation velocity potential. Korkut, E et al (2005) carried out measurements of global loads acting on a Ro-Ro model in regular waves for intact and damaged conditions. The stationary model was tested in different wave heights and wave frequencies for the head, beam and stern quartering seas in order to explore the effect of damages and wave heights on the global loads acting on the model. The analysis of the result indicated that the damages had an adverse effect on the loading conditions on the model depending upon the directionality of the waves and frequency range applied. This effect might cause structural damage on the ship and danger the safety of the ship and passenger on board. Kim, D.H and Paik, J.K (2017) developed a fully automated methodology for the optimum design of hull structural scantlings for merchant cargo ships that were modelled by plate-shell finite elements. A full optimization technique with multi-objectives was applied for minimizing structural weight and maximizing structural safety, as per design constraints associated with the ultimate limit states of the plate panels, support members and hull girders. The developed procedure was applied to the hull structural scantlings of a very large crude oil carrier (VLCC), and the test demonstrated the procedure's capacity to meet the strength requirements of common structural rules. Muis Alie, M.Z et al (2016) investigate the influence of superstructure on the longitudinal ultimate strength of a Ro-Ro ship. To investigate the ultimate strength, the Smith's method was adopted and implemented into the thin-walled beam. The cross section of Ro-Ro ship was taken to be analyzed. Muis Alie, M.Z et al (2017) assessed the ultimate hull girder strength of Ro-Ro ship after damaged. The cross section of Ro-Ro was taken to be analyzed. The collision and grounding damages were assumed to be placed on the side and bottom area. The damages were created by removing the element from the side shell and bottom parts. Finally, the result obtained was compared with one another. Also, the progressive collapse analysis of ship hull girder based on Smith's method was developed by Yao and Nikolov (1992). Naar, H et al (2004) described a couple beam method, which estimate elastic response in the longitudinal bending of a passenger ship with a large multi-deck superstructure. The method could be applied during an early project stage, when detailed three-dimensional finite element modelling was not yet possible. The theory was based on assumption that each deck in the superstructure and also the main hull could be considered as thin-

walled beam.

In the present study, the ultimate hull girder strength analysis is assessed considering the section modulus. The cross section of a Ro-Ro ship is considered to be analyzed. Bottom part, car and passenger decks are calculated for the section modulus where those located above and/or under the neutral axis. Their result is investigated toward the ultimate strength for the global structure.

## ANALYTICAL SOLUTION

The section modulus of ship indicates the ship strength not only in longitudinal but also transversal direction. The classification societies have been stated requirements so that the section modulus should be greater than a prescribed value. It is well known that ship structural characteristics affect significantly on the ultimate strength depending on the cargo types, configuration of structural scantling and so on. The fundamental theory of strength of material may be used for calculating the section modulus of the ship hull cross section. In structural modelling, the ship hull cross section is idealized by stiffened and unstiffened plate combination. Fig. 1 shows the cross section of Ro-Ro ship and Fig. 2 the typical type of stiffened plate with attached plating.

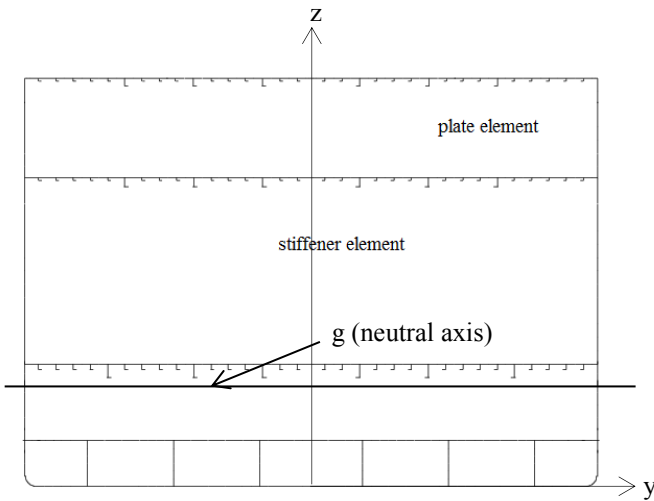


Fig. 1 Cross section of Ro-Ro ship

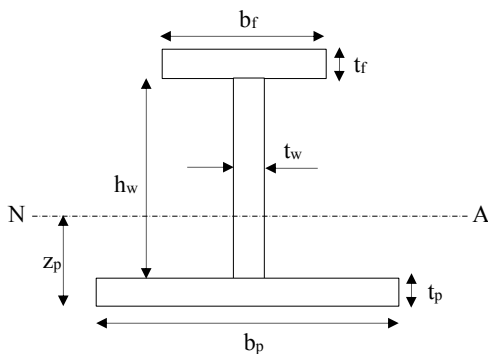


Fig. 2 Typical type of stiffened plate with attached plating

The profile and consists of web, flange and attached plating including the neutral axis (N-A) are shown in Fig. 2. The moment of inertia of the

profile may be expressed as

$$I = \frac{b_p(t_p)^3}{12} + A_p \left( z_p - \frac{t_p}{2} \right)^2 + \frac{h_w^3 t_w}{12} + A_w \left( z_p - t_p - \frac{h_w}{2} \right)^2 + \frac{b_f t_f^3}{12} + A_f \left( t_p + h_w + \frac{t_f}{2} - z_p \right)^2 \quad (1)$$

Where

$$A_p = b_p t_p \quad (2)$$

$$A_w = h_w t_w \quad (3)$$

$$A_f = b_f t_f \quad (4)$$

$$z_p = \frac{0.5 b_p t_p^2 + A_w (t_p + 0.5 h_w) + A_f (t_p + h_w + 0.5 t_f)}{(A_p + A_w + A_f)} \quad (5)$$

The location of the neutral axis,  $g$ , of the full cross section as shown in Fig. 1 above the base line can be obtained by assuming that all longitudinal strength elements are fully effective, those are

$$g = \frac{\sum A_i z_i}{\sum A_i} \quad (6)$$

where  $A_i$  is the cross-sectional area of the  $i$  th plate stiffener element with fully attached plating and  $z_i$  is the coordinate of the  $i$ -th element,  $z = 0$  is taken at the base line. The moment of inertia of the hull cross section is calculated by the following formula

$$I_y = \sum A_i (z_i - g)^2 + \sum i y_i \quad (7)$$

Where  $i y_i$  is the moment of inertia of each element such as stiffener, plate between stiffeners with respect to the neutral axis of each element. The local classification society rules determines the moment of inertia by the following approach

$$I_y = 3 \times 10^{-2} W \frac{L}{k} \quad (8)$$

where  $W$ ,  $L$  and  $k$  are the section modulus, length of ship and material factor (BKI, 2017). According to the formula, the material factor is very important to obtain moment of inertia. The section moduli at the deck and bottom part denoted by  $W_D$  and  $W_B$  are given by

$$W_D = \frac{I_y}{D - g} \quad (9)$$

$$W_B = \frac{I_y}{g} \quad (10)$$

where  $D$  represents as the ship's depth. According to local Classification Society rules, the section modulus related to deck  $W_D$  and bottom  $W_B$ , respectively can be obtained by the following formula

$$W = f_r \frac{|M_T|}{\sigma_p 10^3} \quad (11)$$

where  $f_r$ ,  $M_T$  and  $\sigma_p$  are the factor depending on the degree of deck opening, total bending moment (Nmm) and permissible longitudinal bending stress (N/mm<sup>2</sup>), respectively.

The stress components on deck and bottom part can be obtained by using simple expression as follow,

$$\sigma = \frac{Mg}{I} \quad (12)$$

$$\sigma = \frac{M}{W} \quad (13)$$

where  $M$  is the moment on the deck and/or bottom part in hogging and sagging conditions. Here, Eqs. (12) and (13) can be simply expressed as

$$\frac{Mg}{I} = \frac{M}{W} \quad (14)$$

Here, the section modulus can be obtained as

$$W = \frac{I}{g} \quad (15)$$

In the Smith's method, which is applied in the program code developed by Yao and Nikolov (1992) is used. The explanation is briefly described such as the axial stress  $\sigma_i$  corresponding to the axial strain  $\varepsilon_i$  is given by the average stress-average strain relationship for the individual elements as illustrated in Fig. 3. The average stress-average strain relationship is derived considering of buckling and yielding

$$\sigma = f_i(\varepsilon) \quad (16)$$

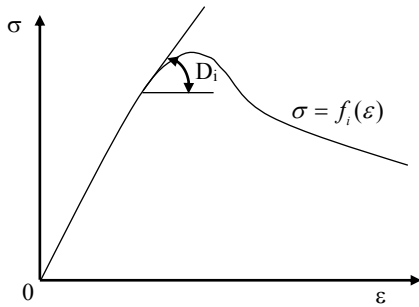


Fig.3 Average stress-average strain relationship for structural element

where  $f_i(0) = 0$ . The axial force  $P$ , the vertical bending moment  $M_V$ , and the horizontal bending moment  $M_H$  can be obtained by integrating axial stresses over the intact part of cross section as

$$P = \sum_{i=1}^N \sigma_i A_i \equiv 0 \quad (17)$$

$$M_H = \sum_{i=1}^N \sigma_i (y_i - g) A_i \quad (18)$$

$$M_V = \sum_{i=1}^N \sigma_i z_i A_i \quad (19)$$

The  $y$  and  $z$  are the coordinates of the cross section measured from the origin at the bottom keel as shown in Fig. 4.

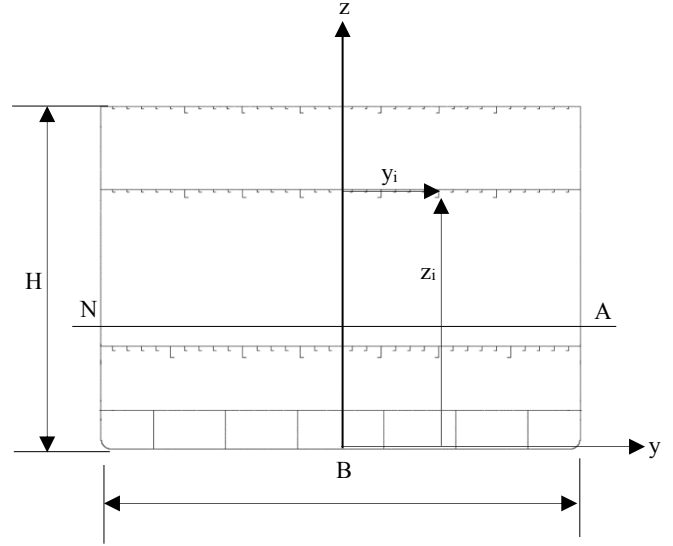


Fig.4 The coordinate systems of the cross section

When the axial load is added to bi-axial bending, the stiffness equation is expressed in term of general formula,

$$\begin{Bmatrix} \Delta P \\ \Delta M_H \\ \Delta M_V \end{Bmatrix} = \begin{bmatrix} D_{AA} & D_{AH} & D_{AV} \\ D_{HA} & D_{HH} & D_{HV} \\ D_{VA} & D_{VH} & D_{VV} \end{bmatrix} \begin{Bmatrix} \Delta \varepsilon \\ \Delta \phi_H \\ \Delta \phi_V \end{Bmatrix} \quad (20)$$

Where

- $\Delta P$  : increment of axial force
- $\Delta M_H$  : increment of horizontal bending moment
- $\Delta M_V$  : increment of vertical bending moment
- $\Delta \varepsilon$  : increment of axial displacement
- $\Delta \phi_H$  : increment of horizontal curvature
- $\Delta \phi_V$  : increment of vertical curvature

and the tangential stiffness of the cross section are written as

$$D_{AA} = \sum_{i=1}^n D_i A_i \quad (21)$$

$$D_{AH} = D_{HA} = \sum_{i=1}^n D_i y_i A_i \quad (22)$$

$$D_{HH} = \sum_{i=1}^n D_i y_i^2 A_i \quad (23)$$

$$D_{AV} = D_{VA} = \sum_{i=1}^n D_i z_i A_i \quad (24)$$

$$D_{VV} = \sum_{i=1}^n D_i z_i^2 A_i \quad (25)$$

$$D_{HV} = D_{VH} = \sum_{i=1}^n D_i y_i z_i A_i \quad (26)$$

## METHODOLOGY

The ultimate strength analysis considering the cross-section modulus of Ro-Ro ship hull girder is performed using analytical formulation. The cross section of Ro-Ro ship is taken to be analyzed. Two Ro-Ro ships, Type-1 and Type-2 are considered as the object ships as shown in Table 1. Both of them are designed based on the local Classification Society rules as shown in Figs. 5 and 6.

Table 1 Ship dimensions

Ro-Ro Ship	Type-1	Type-2
L (mm)	65000	50500
B (mm)	15000	14000
D (mm)	10693	10950

The Ro-Ro ships consist of three decks, which are car, passenger and top decks. The differences between of type-1 and type-2 Ro-Ros are number and dimension of the stiffeners, number of cars and passengers and the configuration of the structural shape particularly in the bottom part. Type-2 Ro-Ro is deeper than type-1 Ro-Ro, while type-1 Ro-Ro is wider than type-2 Ro-Ro. One-frame space is considered in the longitudinal direction. The material properties such as young's modulus and yield strength are related to the ship's characteristics, while poisson's ratio is set to be constant. The initial imperfection, welding residual stress, damage, and crack are not considered in the analysis. The ultimate strength is calculated for the intact only in hogging and sagging conditions. It should be noted that there are no longitudinal stiffeners in the bottom of Ro-Ros. Only floors in transversal direction are placed on it. The average stress-strain relationship of each element is derived considering buckling and yielding and integrated to the cross-section.

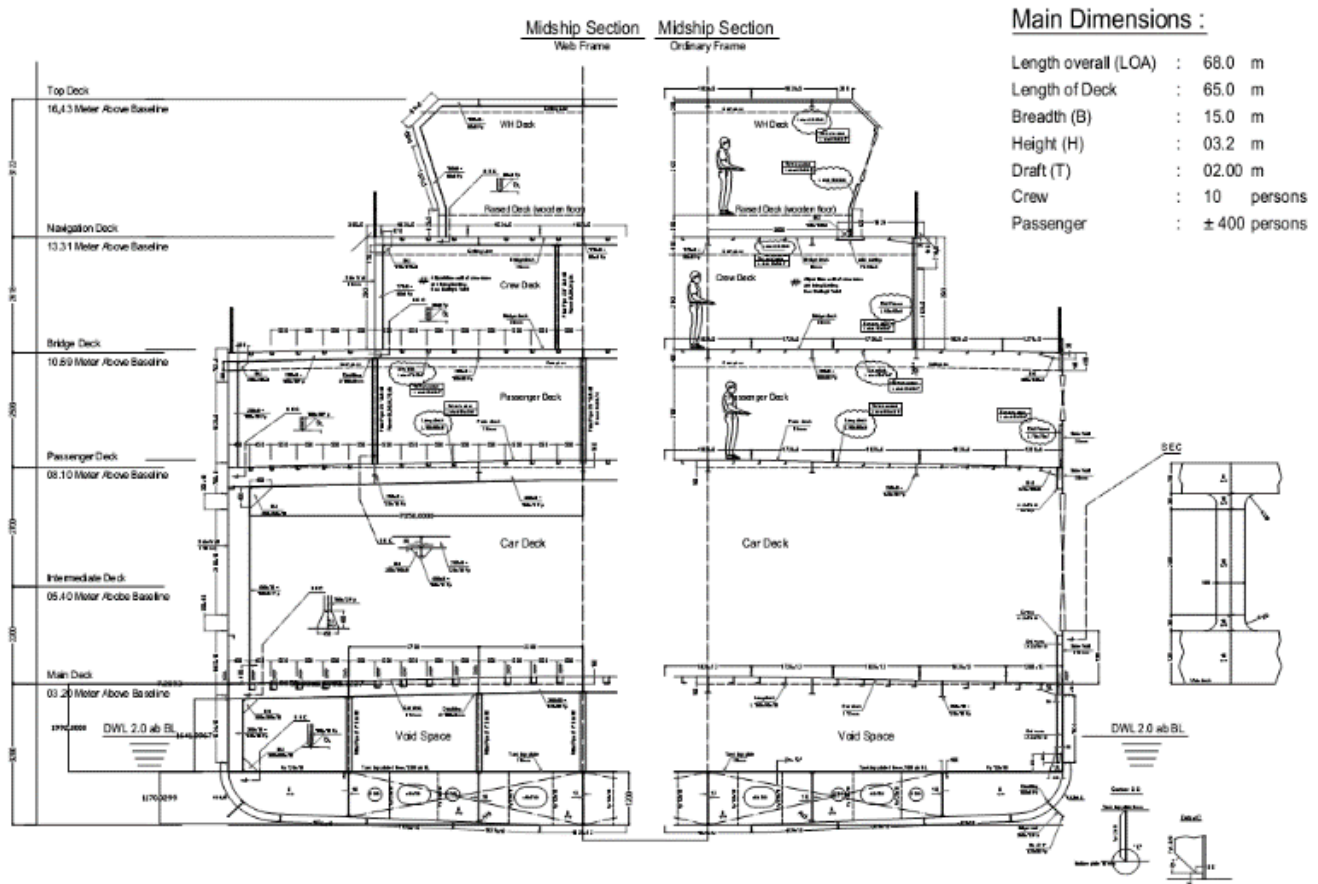


Fig. 5 Ro-Ro type-1

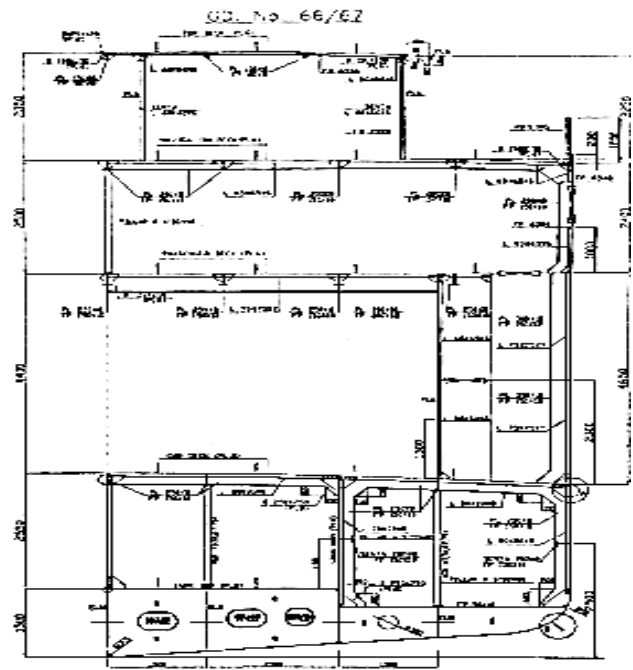
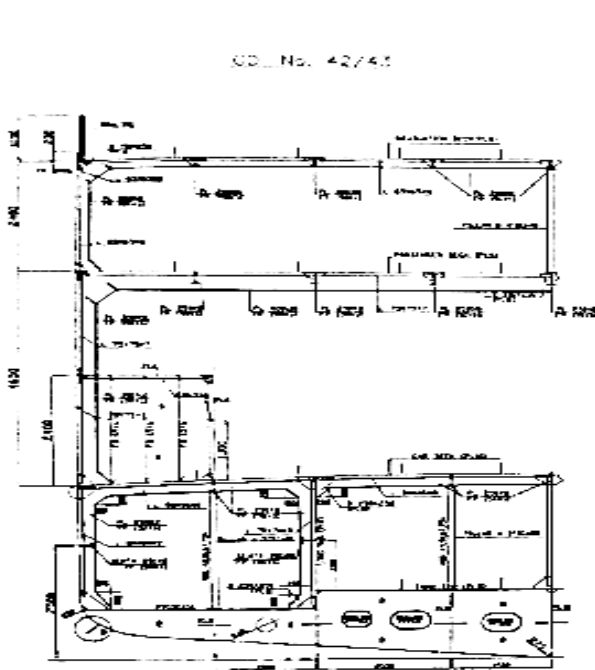


Fig. 6 Ro-Ro type-2

## RESULTS AND DISCUSSIONS

The ultimate strengths of Ro-Ro ships considering their section modulus are confirmed applying the method developed by Yao and Nikolov (1992) according to Smith's method. In this case, the incremental iterative approach is adopted to obtain the ultimate strength both under hogging and sagging condition. In the Smith's method, the cross section is divided into the elements composed of the stiffened and unstiffened plates. The cross section is assumed to be remained plane during progressive collapse and there is no interaction between adjacent elements in the cross section. The vertical bending moment is applied at both sides of the cross section. The cross section of the ships are illustrated in Figs. 7 and 8, respectively. The navigation and upper deck of Ro-Ro ships are assumed to be eliminated.

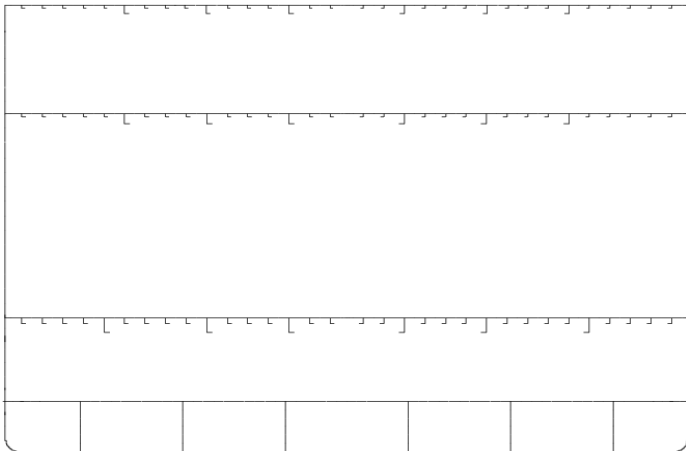


Fig. 7 Cross section of Ro-Ro type-1

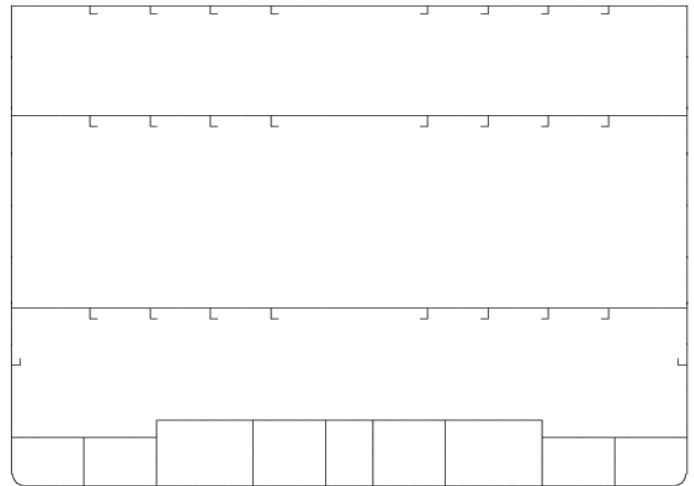


Fig. 8 Cross section of Ro-Ro type-2

The hull girder cross section is very sensitive leading to their ultimate strength and behaviour. It is well known that there is relationship between stress, moment, moment of inertia, neutral axis and section modulus. These relationships are expressed in Eqs. 12, 13, 14 and 15. This is why section modulus depends on bending moment and the section modulus can be obtained by calculating the elements. The corresponding section modulus to the Ro-Ro ship types is shown in Fig. 9. It is observed that the section modulus for type-2 is smaller than type-1. The characteristics of the both ship are completely different starting from the bottom part to the deck. However, two ships consist of three decks, but the shape configuration including number and dimension of stiffeners and double bottom structure also give significant influence to the section modulus and their ultimate strength.

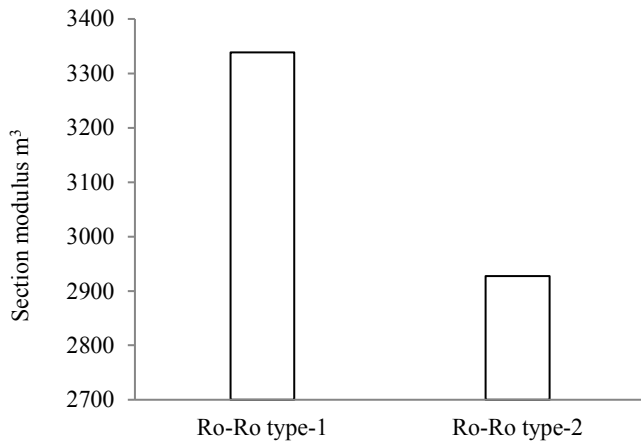


Fig. 9 Section modulus of Ro-Ro ships

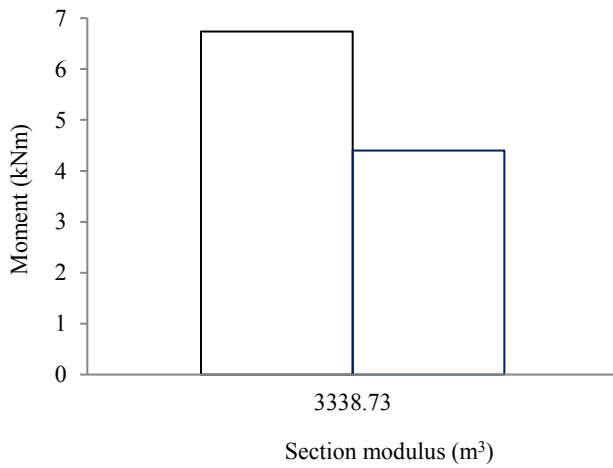


Fig. 10 Moment-section modulus for Ro-Ro type-1

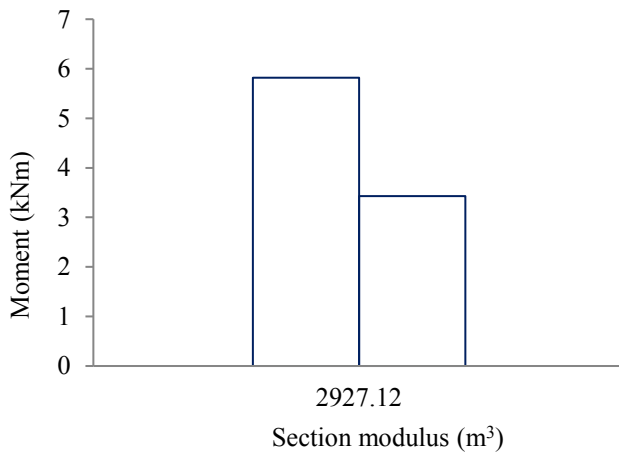


Fig. 11 Moment-section modulus for Ro-Ro type-2

Table 2 Moment-section modulus relationship for hogging

Ship types	M hogging (kNm)	Section modulus (m <sup>3</sup> )
Roro type-1	6.74	3338.73
Roro type-2	5.82	2927.12

Table 3 Moment-section modulus relationship for sagging

Ship types	M sagging (kNm)	Section modulus (m <sup>3</sup> )
Roro type-1	4.40	3338.73
Roro type-2	3.43	2927.12

The relationship between moment and section modulus of two Ro-Ro ships are described in Figs. 10 and 11 in hogging and sagging conditions, respectively. The moment-section modulus relationship for type-1 is also larger than for type-2 in hogging and sagging condition as shown in tables 2 and 3. The shapes of the cross section for two ships are quite different especially for type-2, because type-2 has double bottom which is not straight line where those area consists of some pillars. The pillars attached on the vertical plate to overcome the structural deformation at the car deck.

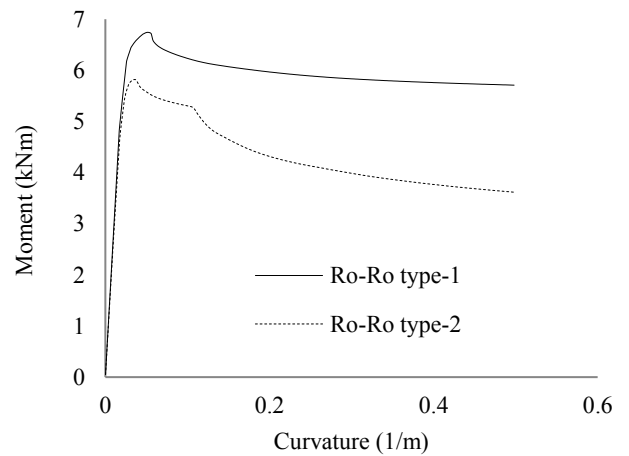


Fig. 12 Moment-curvature relationship for hogging

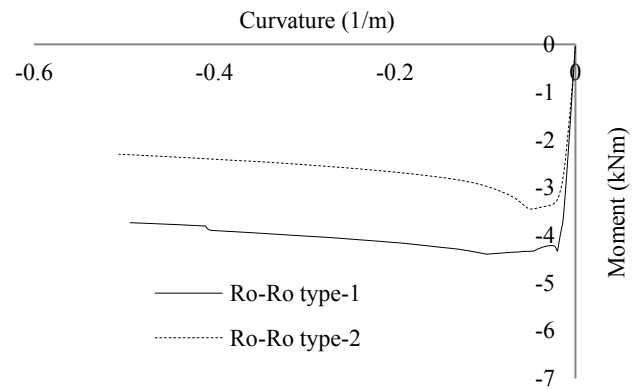


Fig. 13 Moment-curvature relationship for sagging

The moment-curvature relationship in hogging and sagging conditions for Ro-Ro type-1 and type-2 are described in Figs. 12 and 13. The solid lines express the moment-curvature relationship for Ro-Ro type-1, while the dashed one illustrates for Ro-Ro type-2. The moment-curvature relationship for hogging condition between type-1 and type-2 gives significant differences for the ultimate strength and beyond the ultimate strength. This phenomenon also occurs for sagging condition. The value of the ultimate strength is almost identical with the local Classification Society rules.

## CONCLUSIONS

The ultimate hull girder strength analysis considering section modulus of Ro-Ro ship under longitudinal bending have been performed based on simple formula of the local Classification Society rules and the Smith's method. The following conclusions are; the effect of the section modulus on the ultimate hull girder strength is significant not only in hogging but also sagging condition. The effect may be caused by the structural configuration of the Ro-Ro ships such as dimensions and number of plate and unstiffened plate, especially on the bottom part to support car deck by some pillars.

## REFERENCES

- Biro Klasifikasi Indonesia (BKI). (2017). "Rules for the Classification and Construction," Volume II.
- Kim, D.H and Paik, J.K. (2017). "Ultimate Limit State-Based Multi-Objective Optimum Design Technology for Hull Structural Scantlings of Merchant Cargo Ships," *Ocean Engineering*, 129, 318-334.
- Korkut, E, Atlar, M and Incecik, A (2005). "An Experimental Study of Global Loads Acting on an Intact and Damaged Ro-Ro Ship Model," *Ocean Engineering*, 32, 1370-1403.
- Kukkanen, T and Matusiak, J (2014). "Nonlinear Hull Girder Loads of a RoPax Ship," *Ocean Engineering*, 75, 1-14.
- Naar, H, Varsta, P and Kujala, P (2004). "A Theory of Cupled Beams for Strength Assessment of Passenger Ships," *Marine Structures*, 17, 590-611.
- Muis Alie, M.Z, Sitepu, G, Juswan, Wahyuddin, Nugraha, A.M and Alamsyah (2016). "The Influence of Superstructure on the Longitudinal Ultimate Strength of a RO-RO Ship," *Proc 26th Int Offshore and Polar Eng Conf*, Greece, ISOPE, 4, 1022-1029.
- Muis Alie, M.Z, Sitepu, G and Latumahina, S.I (2017). "The Assesment of the Ultimate Hull Girder Strength of Ro-Ro Ship after Damages," *Proc 27th Int Offshore and Polar Eng Conf*, USA, ISOPE, 4, 913-919.
- Yao, T. and Nikolov, P.I. (1992). "Progressive Collapse Analysis of a Ship's Hull Girder under Longitudinal Bending (2nd Report)," *J Soc. Naval Arch. of Japan*, Vol 172, 437-446.